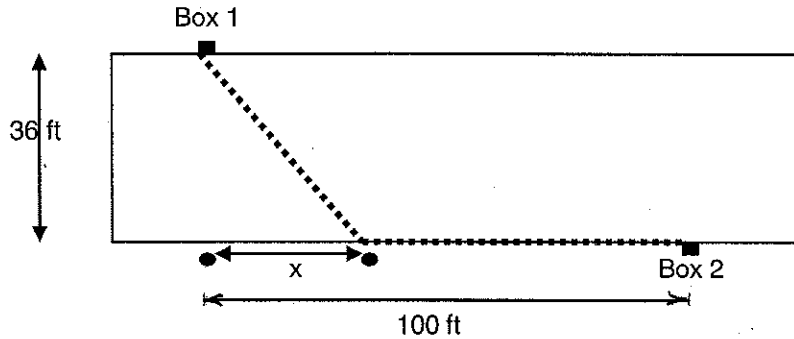


#25 Optimization Problems 4.4

1. A square sheet of tin 2 meters on a side is to be used to make an open-top box by cutting a small square of tin from each corner and bending up the sides. How large a square should be cut from each corner in order that the box have as large a volume as possible?

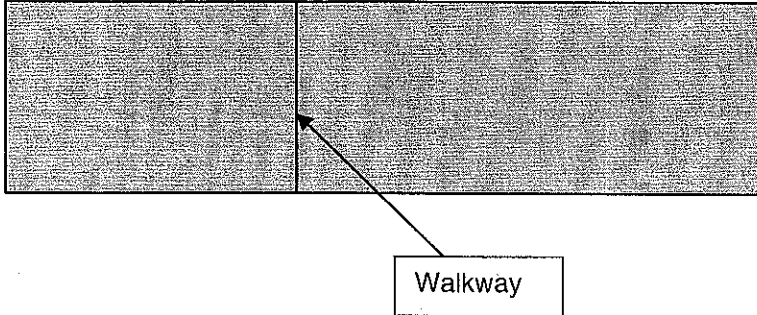
- (a) What is to be optimized?
- (b) Make sketches. What varies? How are they related? Label your sketches clearly by assigning variables to quantities which vary.
- (c) Obtain a formula for the function to be optimized in terms of the variables that you identified in part (b). Choose one variable and write the others in terms of this one. What is the domain of this variable?
NOTE: MAX/MIN PROBLEMS CAN BE SOLVED BY USING IMPLICIT DIFFERENTIATION WITH MORE THAN ONE VARIABLE.
- (d) Find the critical points.
- (e) Use either the 1st derivative test or the 2nd derivative test to establish whether each critical point is a local max, a local min or neither.
- (f) Evaluate the function at these points and at the endpoints of the domain to find the global maxima or minima.
- (g) Check to be sure that you have answered the question asked for in the problem

2. A telephone installation crew must run a line underground between two junction boxes. Unfortunately there is a 36 ft wide paved road between the two boxes, and one box is 100 feet down that lane from the other. It costs \$30 per foot to cut and repair the paved road, but only \$24 per foot to dig and refill along the side of the road. The crew will cut and repair the paved road to a point x feet from the point directly across from the first junction box, and then dig along the road the rest of the way. They are told that they should aim for a point $x = 48$ ft from the point directly across from the junction box. Show that $x = 48$ is the point where the cost of the line will be the least possible.

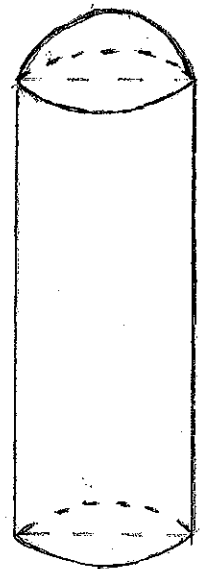


THE 100 FT LINE SHOULD GO FROM THE BEGINNING OF THE "X" SEGMENT TO BOX 2.

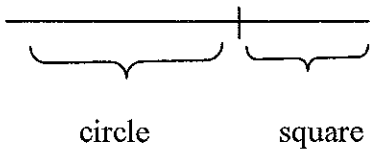
3. An architect designs a rectangular garden. The garden is to be surrounded on all sides by a hedge; a trellised walkway runs between one pair of opposite sides. The hedge costs \$98 per linear foot of length, the walkway costs \$127 per linear foot. The garden must have a total area of 1000 ft^2 . What dimensions should the garden have to minimize the cost?



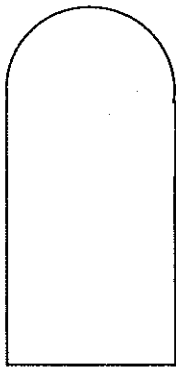
4. A silo is to be made in the form of a cylinder surmounted by a hemisphere. The cost of construction per square foot of surface area is twice as great for the hemisphere as for the cylinder. Determine the dimensions to be used if the volume is fixed and the cost of construction is to be a minimum. Neglect the thickness of the silo and waste in construction. The floor of the silo is not included in this construction.



5. A wire of length 12 inches can be bent into a circle, a square, or cut to make both a circle and a square. How much wire should be used for the circle if the total area enclosed by the figure(s) is to be a minimum? A maximum?



6. A window consisting of a rectangle topped by a semicircle is to have a perimeter P . Find the radius of the semicircle if the area of the window is to be a maximum.



7. A rectangular field as shown is to be bounded by a fence. Find the dimensions of the field with maximum area that can be enclosed with 1000 feet of fencing.

